

Thesis Summary:  
**A Nonprehensile Method for Reliable Parts Orienting**  
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**ABSTRACT**

*Prehension* may be defined as “The act of taking hold, seizing or grasping, as with the hand” (Webster’s 3rd International Dictionary). *Nonprehensile manipulation*, then, can be defined as the manipulation of objects without grasping them. Manipulation without prehension is a natural way of handling objects for both humans and machines. The ability to manipulate objects which may not be graspable increases the flexibility of a robot interacting with its environment, without adding complexity to the mechanical design. This research analyses the mechanics of nonprehensile contact between a simple, two degree of freedom manipulator and a part. The intent is to develop reliable but sensorless manipulation routines for use in an automated assembly environment. While nonprehensile, sensorless devices are in common use in such environments, existing parts orienting devices, such as bowl feeders or the SONY Automatic Parts Orienting System, must be custom designed for each specific task. To decrease the setup or changeover time for an assembly line, what is needed is a simple but more general device, which can be easily modified or reprogrammed in response to a change in tasks. We present a planning algorithm for sensorless parts orienting in the plane with two one degree of freedom palms. Our method finds feasible paths through the space of equivalent state configurations of the object in the palms, without requiring that the palms maintain stable support of the object over the entire path. We show that such a device can reliably orient parts in the plane. Planning reorientations requires only the geometrical descriptions of the parts. The plans produced by our algorithm are robust to uncertainties in the part’s initial state and in the coefficient of friction, as well as to small inaccuracies in manipulator calibration.

# 1 INTRODUCTION

## 1.1 WHY NONPREHENSILE MANIPULATION?

*Prehension* may be defined as “The act of taking hold, seizing or grasping, as with the hand” (Webster’s 3rd International Dictionary). *Nonprehensile manipulation*, then, can be defined as the manipulation of objects without grasping them. Manipulation without prehension is a natural way of handling objects for both humans and machines. The ability to manipulate objects which may not be graspable increases the flexibility of a robot interacting with its environment, without adding complexity to the mechanical design. Nonprehensile manipulation also forms an intermediate point in the range of manipulation operations. At one end of this range we have operations for which complete control is necessary or desired over the complete duration of the operation: either complete control over the object, or complete control over the object’s interactions with the environment. In between these extremes lies a large class of operations during which complete constraint over the object to be manipulated is either undesirable or impractical, but some control over the object is desired over its entire trajectory, in order to bring the object reliably to a desired final state. Moving objects which are too large to be grasped, but must be pushed or rolled to a desired location, is one example. Other examples include shutting a drawer, or pushing a door shut without slamming it. Another example is coin sorting. One way to sort coins is to pick each coin up individually, determine the denomination of the coin, and place it into the appropriate stack. A better way is to dump all the coins into a device which can mechanically filter and sort the coins by their size and weight. We claim that nonprehensile manipulation is appropriate for exactly these type of tasks. Nonprehensile manipulation is also appropriate in many situations where, rather than force-closure, it is sufficient to ensure that the contact forces resist a specific external wrench. We will call this *stable support* [1].

The examples of nonprehensile manipulation given above range from very large ungraspable objects at one end, to many small objects at the other. This work concerns itself with attempting to reliably and quickly orient small objects, although some results may be useful in other domains. Directions of interest include (i) controlling the shape of constraint surfaces of systems in such a way that constraint and external forces naturally attract the system to the desired state, and (ii) exploiting the system dynamics in a similar fashion, much as in the research on underactuated manipulators. *Palmar manipulation* — the manipulation of an object by its interactions with flat, nearly rigid manipulators — offers a simple domain in which to explore these two directions, although we will mainly concentrate on the first. The work described in this dissertation follows a similar method to Trinkle, *et.al.* ([6], [4]). Their analysis and planning uses the idea of *contact formations* originally presented by Desai, and incorporated into a planner for dextrous manipulation by Trinkle and Hunter [5]. However, the paths through configurations space by [6] and [5] are apparently constrained to always correspond to stable grasps, whereas the method described in our work allows very small unstable motions during the transition from state to state.

We use a manipulator consisting of two one degree of freedom palms modeled as a cone connected at a central hinge. This is a simple example that still captures some of the basic operations of a general two palm system. For this particular example, we show that two basic operations into which manipulations can be decomposed fall naturally out of the equivalence region analysis described above. We believe that similar decompositions will fall out of the analysis of other types of two palm systems.

When we move beyond the quasistatic domain, inertial forces and collision forces begin to matter. Coriolis and centrifugal forces must also be taken into account, so the object does not fly out of the palms, or out of the desired range of positions during the manipulation. For example, one would like to have the object make contact with the palms in such a way that the object does not bounce away from the expected stable position. Although we do not specifically derive velocity constraints, we give qualitative examples of the sort of velocity effects most common to this system.

## 2 MANIPULATION WITHOUT GRASPING

In Chapter 3, we study the problem of reorienting a planar polygonal part under the following assumptions:

- We will restrict ourselves to planar polygonal objects, although our methods should carry over without much difficulty to any planar object whose convex hull has a finite number of stable resting positions. Cylindrical objects can also be modeled as planar.

- Force balance is achieved by the palms stably supporting the object against a known gravitational force. No other external forces are considered, hence complete force/form closure is not necessary.
- We will assume that the motions of the manipulator are slow compared to gravity, so that the kinetic energy imparted to the object by the motion of the palm is dominated by the object’s potential energy.
- We will assume that the contacts between the object and the palms are very low friction (*i.e.*, the contacts are all sliding), so that we may approximate the system with a frictionless analysis.
- We will model the two palms as a “cone” manipulator: two palms connected at a central hinge. This is a simple model that still captures many of the basic operations of a general two palm system.

In later chapters, we show how to relax the assumptions of low friction and low kinetic energy.

## 2.1 ENERGY ANALYSIS

The motion of a polygonal object when in contact with the two palms can be determined by constructing the configuration space obstacle formed by the palms. Each palm generates an obstacle in configuration space, and the intersection of the surfaces of these two obstacles form a curve describing the poses of the part when it is making contact with both palms simultaneously. We can use the potential energy of the part to identify its *equilibrium* positions.

We show that if the cone is frictionless, then all stable equilibrium positions of the part in the palms correspond to an edge of the part making contact with one of the palms. Hence, assuming that the palms move slowly in comparison with gravity (so that kinetic energy is low), an object caught in a cone in a particular orientation will settle to a unique resting position determined by the initial position of the object upon contact, and the tilt of the cone with respect to gravity.

We would like to use the above observations to plan object reorientations automatically. We will call all stable resting configurations that correspond to a particular side of the object in edge contact with a particular palm *equivalent configurations*. For example, all stable configurations where the edge  $e_a$  of some part rests on the left palm are equivalent. An equivalence region can be described by an edge of a part, and the palm with which this edge makes contact. In the state space of the system, parameterized by the part orientation, relative angle between the palms, and the orientation of the cone formed by the palms in the world frame, an equivalence region is planar. Define a *stable edge* of a part as an edge  $e_s$  such that, if one were to project the center of gravity onto  $e_s$ , this projection would lie in the interior of the line segment  $e_s$ . Equivalence regions corresponding to stable edges of the part are simply connected. For every equivalence region there is also a *preimage* in the state space: those regions of the state space for which the part is trapped in a potential energy well, whose minimum corresponds to a point in the equivalence region. The boundaries of an equivalence region correspond to the edge of stability of a particular contact class,  $\mathcal{S}_0$ : there will be some directions of movement of the cone which will take the system state out of  $\mathcal{S}_0$ , causing the edge contact of interest to be lost. If that particular boundary region of  $\mathcal{S}_0$  lies in the preimage of another equivalence class,  $\mathcal{S}_1$ , then the object will fall into the stable contact corresponding to equivalence region  $\mathcal{S}_1$ . In other words, an object’s orientation can be brought from  $\mathcal{S}_0$  to  $\mathcal{S}_1$  by bringing the object to the appropriate boundary of  $\mathcal{S}_0$  and moving the cone in such a way that the system state moves out of  $\mathcal{S}_0$ , and into the preimage of  $\mathcal{S}_1$ . This transition is reliable even though the manipulator does not maintain stable support of the object during the transition from  $\mathcal{S}_0$  to  $\mathcal{S}_1$ , as long as the object’s kinetic energy is low compared to its depth in the potential energy well.

## 2.2 PLANNING IN THE FRICTIONLESS DOMAIN

In Chapters 3 and 7, we prove the following results about planning reorientations in the frictionless domain.

First we define two elementary palm motions. Let the relative angle between the palms be  $\phi$ , and the orientation of the cone in the world frame be  $\beta$ . We define a *pure tilt* motion as one where  $\phi$  is held constant as  $\beta$  varies. We define a *fixed- $\theta$  squeeze* as a motion where the palm making edge contact with the object stays fixed, and the other palm opens and closes, resulting in the cone configuration changing, but the object orientation relative to the world frame remaining fixed as long as the state is stable. In the parameter space, this corresponds to  $\frac{\partial\beta}{\partial\phi} = \varepsilon\frac{1}{2}$ , where  $\varepsilon = -1$  for the left palm held fixed, 1 for the right palm held fixed.

**Theorem 1** Given any two points in an equivalence region  $\mathcal{S}$  corresponding to a stable edge of a polygon  $P$ , there exists a path entirely contained in  $\mathcal{S}$ . which can be decomposed into at most two tilts and one fixed- $\theta$  squeeze.

If the equivalence region does not correspond to a stable edge, but the two points are in the same component of the equivalence region, there still exists a path between the two points composed only of pure tilts and fixed- $\theta$  squeezes which is entirely contained in that component.

Therefore, if there exists a path from an arbitrary start state to an arbitrary end state, where the transitions between equivalence regions are given by pure tilt motions, the entire path between the start state and the goal state can be decomposed naturally into pure tilts and fixed- $\theta$  squeezes.

Intuitively, pure tilts roughly correspond to pure rotations of the objects, and fixed- $\theta$  squeezes roughly correspond to pure radial translation. Just as pure rotations and pure radial translation span the plane when using polar coordinates, pure tilts and pure rotations span the motion space of the object in the palms.

In addition to moving around within a stable equivalence region, we also would like to know what motion will take us to a new equivalence region. In order to determine which orientations of a particular part can be brought to which other orientations, first determine all the equivalence regions, (two for every flat face of the convex hull of the object) and their preimages. Second, determine the boundary of each equivalence region, and divide each boundary into segments, according to which preimage of another equivalence class that segment is contained in. Third, construct the graph  $\mathcal{G}$  whose nodes are the equivalence regions, with arcs denoting which equivalence regions transit into another. Each arc is labeled with the appropriate set of cone configurations (for the example above, the arcs would be labeled by the appropriate  $\phi$ -interval), and the direction in cone configuration space in which the cone must be moved. Figure 1 shows  $\mathcal{G}$  for our example object. The arcs in  $\mathcal{G}$  were determined by using only pure tilts at the equivalence region boundaries to transit from region to region.

The graph  $\mathcal{G}$  in Figure 1 was generated assuming that all configurations  $(\phi, \beta)$  are achievable. In reality, not all cone tilts and orientations will be achievable, because the finite length of the palms will prevent the object from being held in some cone configurations, and because of other physical limitations of the device. One possible limitation is the ability or inability to slide the part from one palm to the other with out changing the part's orientation relative to the palm on which it rests; we will refer to such a motion as a *sliding transfer*. Sliding transfers may not be possible on a specific device because of the space between the palms, or because of an obstruction (such as the joint) at the vertex of the cone formed by the palms. Such physical limitations can cause certain arcs of  $\mathcal{G}$  to be completely eliminated.

The planning problem has now been segmented into two parts. Given the initial and desired final configurations of the system, the high level problem is how to get from the initial to the final equivalence region. This can be determined by straightforward graph search on  $\mathcal{G}$ . If a path through the graph exists, the reorientation is in principle possible, and the path determines a series of sets of equivalence regions which the cone trajectory must go through.

Once it has been established that a high level path exists, the lower level trajectory planning problem for each equivalence region (node) is to determine the trajectory which the cone must follow to reorient the part. The motions to transit from one equivalence region to another are given by the arcs of  $\mathcal{G}$ . To determine trajectories through equivalent regions, we can take advantage of the fact that equivalence regions are piecewise straight-line connected, as described in Theorem 1. Figure 2 shows an example reorientation for our example object.

Sufficient conditions for the existence of plans are summarized below. We will call a polygon *orientable* if it can be brought from any equivalence region associated with a stable edge, to any other equivalence region associated with a stable edge.

The *relative angle*  $\alpha_{ij}$  between two edges  $e_i$  and  $e_j$  is the counterclockwise angle formed between  $e_i$  and  $e_j$ .

**Theorem 2** Let  $P$  be a convex polygon with  $N$  all stable edges,  $\mathcal{G}$  the corresponding transition graph. In order for  $P$  to be orientable, it is sufficient that either:

1. Sliding transfers be possible, or

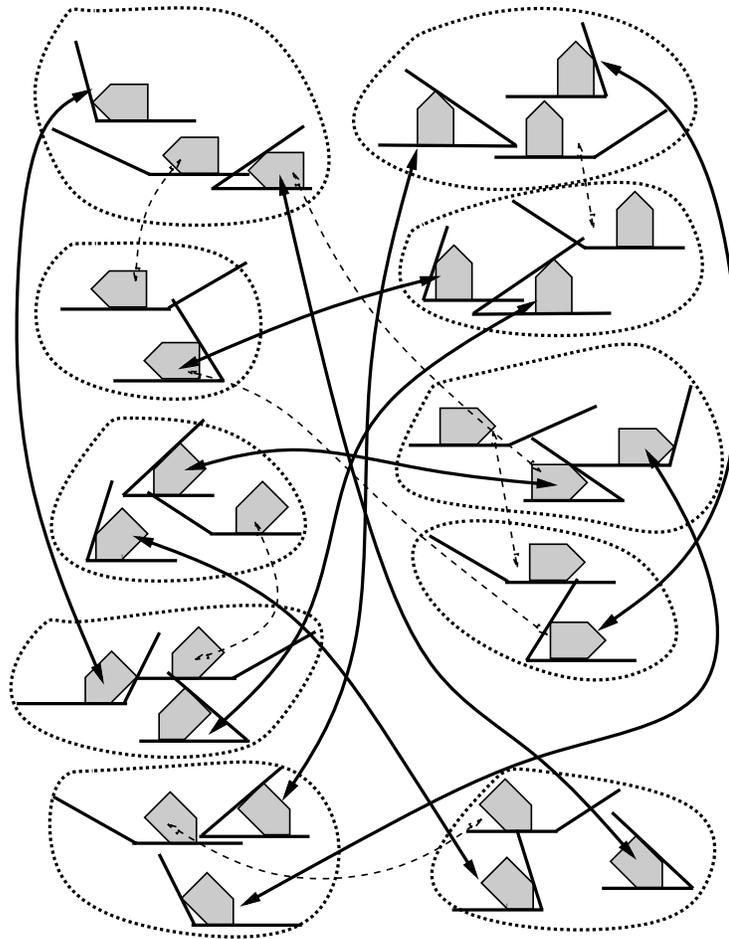


Figure 1: Transition graph for example object. Transition motions are made by pure tilts in the appropriate direction. The arcs shown as dashed lines correspond to transitions which were eliminated by the planner as being physically infeasible for our specific manipulator.

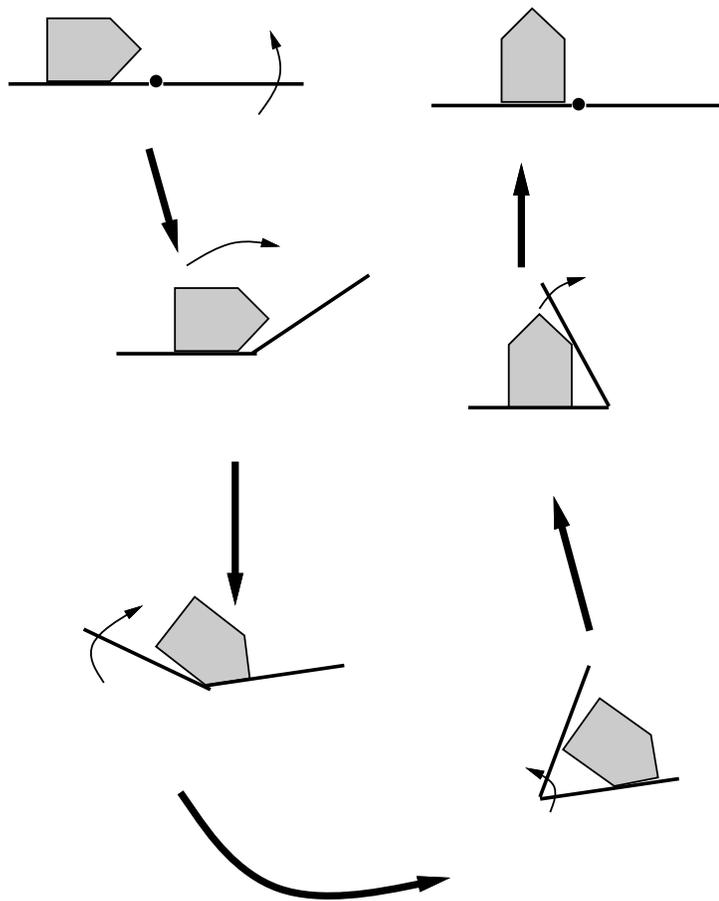


Figure 2: Example reorientation: from  $\theta = -\pi/2$  on left palm to  $\theta = 0$  on left palm



Figure 3: Plastic cone manipulator used to test plans

2. For every edge  $e_i$  of  $P$ , there be at least two edges  $e_j, e_k$  such that  $\alpha_{ij}, \alpha_{ik} \leq \pi$ .

If either of the above two cases is true, then the length of any reorientation plan is bounded by  $N$ . This plan can be found in  $\mathcal{O}(N^2)$  operations.

**Theorem 3** *If sliding transfers are possible, any polygon  $P$  is orientable. The path length from any initial state to any goal state is bounded by  $2M$ , where  $M$  is the number of stable edges. It can be found in  $\mathcal{O}(M^2)$  operations.*

*If sliding transfers are not possible, in order for  $P$  to be orientable it is sufficient that every stable edge  $e_{\alpha_i}$ , when resting on the left palm, can reach two other stable edges  $e_{\alpha_{i+1}}$  and  $e_{\alpha_{i+2}}$  on the right palm by a clockwise tilt. If  $P$  has  $M$  stable edges, the path from any initial state to any goal is bounded by  $2M$  and can be found in at most  $\mathcal{O}(M^2)$  operations.*

### 2.3 EXPERIMENTAL RESULTS

The preceding algorithm was implemented in C on a Dec-station 5000/20. For the example object, the transition graph  $\mathcal{G}$  (Figure 1) can be generated in about one minute. Once  $\mathcal{G}$  is generated, reorientation plans can be found in one or two seconds. Plans were generated to bring the object from the initial stable orientation on a flat palm, to the goal stable orientation on the goal palm, much as in Figure 2. The plans were tested both in simulation and on a plastic cone manipulator (Figure 3), mounted on a tilted air table to reduce support friction. For the example object, the plans simulated tended to either be “robust” to friction as high as about  $\mu = 0.25$ , or be extremely sensitive to the frictionless approximation, failing for friction higher than  $\mu = 0.02$ . Simulation of the plan shown in Figure 2 showed that a static coefficient of friction  $\mu \leq 0.25$  would permit enough of the contacts to slide for the predictions from the frictionless approximation to be valid, and for the plan to succeed.

In the experiments conducted on the air table, the static coefficient of friction was approximately 0.19, low enough for the “robust” plans to succeed. To evaluate the reliability of the example plan, we ran 50 trials, starting the object in its initial orientation,  $\phi = -\pi/2$ , at different arbitrary locations on the left palm. The varying initial configuration of the part led to variation of the part’s trajectory through its configuration space, as expected. Nonetheless, of the 50 attempts, the manipulator failed to correctly reorient the object only 4 times. Despite the variation in the object’s trajectory, the object always stayed trapped in the correct region of the state space and hence would be propelled along to the correct final orientation. Each of the failures seemed to be due to a single rough spot on the right palm, which caused a contact to roll rather than slide.

### 3 FRICTION

Empirically, some edges which are considered stable in the frictionless analysis may be unstable in practice, in the sense that the object may not stay on that edge even when the palms make motions which stay within the corresponding equivalence class. This may be because the object has sufficient energy to escape its energy well, or it may be because friction causes a rolling contact where sliding was expected. In Chapter 4, we evaluate the stability of edges with respect to their tendency to roll rather than slide.

Suppose we have a stable edge  $e_s$ , and its associated equivalence region,  $\mathcal{S}$ , when resting on palm  $k$ . We will say  $e_s$  is *frictionally unstable on palm  $k$*  if there are trajectories through  $(\theta, \phi, \beta)$  space which are contained in  $\mathcal{S}$ , but which fail in practice because of frictional effects: contacts roll which are expected to slide. After identifying frictionally unstable edges, the planner can be augmented with this information, in order to take frictional effects into account.

There is a fundamental tradeoff between the completeness of a given model of the world and its complexity. Because in this work we are interested in designing a planner which is fast and relatively simple to use, in terms of the number and type of its input parameters, we must choose computational simplicity over completeness. The extent to which we consider friction in our world model is far from complete. We consider only two point contact (that is, contact with the resting palm, ignoring the other palm) when looking for rolling contact points, and so will be unable to detect jamming or wedging. We attempt to justify this on two grounds. The first, as we have mentioned, is computational efficiency. From an offline computation point of view, we will only consider the part's resting edge and the resting palm (the two pieces of information which define an equivalence class), rather than consider all possible contact configurations, which gives us  $2N$  cases to consider, rather than  $\mathcal{O}(N^2)$ . From an online computation point of view, we do not wish the overhead of sensors, so we must forgo exact knowledge of state; in particular, whether we are actually in two or three point contact. One can argue that this type of sensory information is binary in nature, and low overhead — a valid argument — but by not having it, we can also dispense with those model parameters which would be required to carry this information around. A lower parameter model of the world is more tractable, and naturally faster to compute. In our case, the frictional model is linear in the number of stable edges, rather than polynomial. We must distinguish between two and three point contact if the difference in these states crucially affects a decision which the planner must make. If in fact the decision can be made independently of this information, and still be a correct decision, then the information is not necessary to the planner.

The second point is that this manipulator technique fundamentally relies on low frictional contact in order to reliably reach potential energy minima without jamming. The potential energy based method of finding stable states is both easy to compute and robust. A minimal energy state in a frictionless world is still a minimal energy state in the presence of friction. An object which is stably supported in the absence of friction will still be stably supported in its presence. A grasp or stable support which is predicted using a particular coefficient of friction, on the other hand, may not be stable if the actual coefficient of friction differs from that of the model. Hence, it can be brittle in the face of model uncertainty. Furthermore, because the location of minimal energy states varies smoothly (linearly) with the palm positions (as long as we remain inside a single equivalence region), the frictionless approach is also robust to small errors in palm calibration. A friction based system may not be. We discuss these points in Chapter 5, when we compare our system to another sensorless two palm system [2], which uses frictional grasps.

#### 3.1 WHEN DOES IT SLIDE AND WHEN DOES IT ROLL?

Referring to Figure 4, we wish to determine the behavior of the object on the palm under the influence of gravity. For what orientations of the palm will the object remain stationary? For orientations of the palm steeper than that, will the object slide down the palm, or roll about a vertex? In Chapter 4, we discuss how to determine the stability of an equivalence class with respect to the contact friction. That is, for each equivalence class, we can find an upper bound on the coefficient of friction such that for values of friction lower than this bound, if the part is in this equivalence class, it will behave as if the palms were frictionless: the contacts will slide, not roll. The results can be summarized as follows.

Suppose we are tilting the palm clockwise (so that the tilt angle of the palm,  $\beta$ , is negative). Then the critical contact is the right vertex of the resting edge. Let  $h$  be the height of the center of gravity, and  $w_r$  be the tangential distance from the center of gravity to the right vertex. Consider an object resting on stable edge

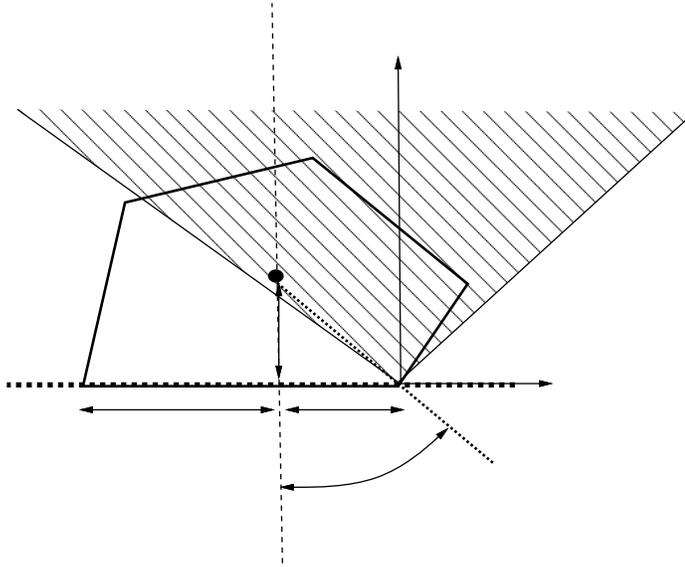


Figure 4: Considering the case of a clockwise tilt ( $\beta < 0$ ).

contact with a single palm. Let  $\mu_{crit} = \frac{w_p}{h}$ , and  $\alpha_{crit} = \arctan \mu_{crit}$ . When the palm is tilted clockwise, one of the following cases occurs:

- If  $\mu < \mu_{crit}$ , the object will stick if  $|\beta| < \arctan \mu$ . For steeper tilt angles, the object will slide without rotating.
- If  $\mu > \mu_{crit}$ , the object will stick if  $|\beta| < \alpha_{crit}$ . For steeper tilt angles, the object will rotate clockwise about the right vertex.

Similar results hold for the left vertex when the palm is tilted counterclockwise.

Using this result, we now have a stability criterion for a particular equivalence class with respect to the contact friction of our system. If the equivalence class corresponds to resting on the left palm, we consider  $\mu_{crit}$  for the right vertex of the resting edge. If the object is resting on the right palm, we consider  $\mu_{crit}$  for the left vertex of the resting edge. In either case, if  $\mu_{crit}$  is less than the system contact friction, the resting edge will have a tendency to roll as the palm tilts out of the horizontal, and the equivalence class may be considered to be frictionally unstable. Frictional instability is determined using only the *direction* of the next tilt motion (which we know), not the *magnitude* of the tilt (which we do not know, until after we have found the plan).

Although this stability criterion cannot be incorporated directly into the transition graph described previously in a useful way, we can use this criterion to augment the transition graph which is described in Chapter 5. This larger transition graph takes into account uncertainty in the state of the world, not only the uncertainty associated with friction, but that of the initial state of the system as well. If a certain motion causes the object to fall into an equivalence class which is frictionally unstable, we consider this motion to have two possible outcomes. One is the equivalence class which is determined by the frictionless analysis. The other is the equivalence class which the object may roll to due to friction. For example, if a certain motion has as one of its possible output states the state  $(e_i, \text{left})$ , and this state is frictionally unstable with respect to the contact friction of the system, then the object in state  $(e_i, \text{left})$  will have a tendency to roll clockwise. The next edge clockwise is edge  $e_{i-1}$ , and so the state  $(e_{i-1}, \text{left})$  should also be considered as a possible output state. This augmentation will not be sufficient if the object has enough energy to roll over past edge  $e_{i-1}$  to another subsequent edge, but is valid under the assumption of low kinetic energy.

By considering both the sliding and the rolling possibilities, the resulting transition graphs will be valid for any coefficient of friction less than the coefficient used to actually construct the transition graph. It will not

be valid for coefficients greater than this, however, since a greater coefficient of friction may cause previously stable edges to become frictionally unstable.

## 4 UNCERTAINTY: PLANNING FROM AN UNKNOWN INITIAL STATE

In many applications, such as parts feeding, the initial state of the object may not be known. If a reorientation plan can be found to reliably bring the object from any initial state to a single known final state, then the method from the previous section can be applied to bring the object from that known state to any desired goal state. In Chapter 5 we focus on the problem of determining a palm trajectory which will always bring the part to a single final state.

### 4.1 FRICTIONLESS, LOW ENERGY CASE

Under the assumptions about the system used in Chapter 3 (all contacts slide, kinetic energy is low), the transition from initial equivalence region to final equivalence region is unique for a given cone opening  $\phi$ . If there are  $2N$  equivalence regions, then we can in principle build the  $2^{2N}$  elements of the *power set* of the equivalence regions: that is, the set of all possible combinations of the equivalence regions. For instance, if we have a set of equivalence regions  $\{A, B, C, D\}$ , then the power set of this set of equivalence regions would be  $\{\{A\}, \{B\}, \{C\}, \{D\}, \{A, B\}, \{A, C\}, \{A, D\}, \dots \{A, B, C, D\}\}$ . Given some set of palm motions, we can build a larger transition graph,  $\mathcal{GG}$  where each node is an element of the power set, and each arc corresponds to a palm motion. Each arc then connects a set of initial states to its corresponding set of final states. For instance, suppose for a given palm motion, if the object was initially in state A, the motion will transfer the system state to C. If the object started in state B, the same palm motion will transfer the system to state D. Then the graph  $\mathcal{GG}$  would include a node corresponding to the set  $\{A\}$  with an arc to a node corresponding to the set  $\{C\}$ , a node corresponding to the set  $\{B\}$  with an arc to a node corresponding to the set  $\{D\}$ , and a node corresponding to the set  $\{A, B\}$  with an arc to a node corresponding to the set  $\{C, D\}$ . All the arcs in this example correspond to the same palm motions. We can then do breadth-first search over  $\mathcal{GG}$ , starting from the set of all possible initial states, in the hope of finding a sequence of arcs that will take the system to a node where only one state is possible. We will call such a sequence of arcs a *homing sequence*.

If we wish to find a homing sequence which homes to a particular final state, we can do a breadth-first backchaining search from our desired final state, hoping to find a path backwards to the set of all possible initial states.

In practice, since an arc will generally correspond to bringing an object from resting on one palm to resting on the other palm, we do not have to consider all  $2^{2N}$  elements of the power set. We will generally have to consider the set of all initial states, all combinations of equivalence regions corresponding to resting on the left palm, and all combinations of equivalence regions corresponding to resting on the right palm (excluding the empty set), for a total of  $2(2^N - 1) + 1 = 2^{(N+1)} - 1$  sets of combinations of states.

Figure 5 shows a homing sequence found for our example object. For each cone opening considered, there were two possible motions: One type of motion considered was to start with the left palm horizontal and the palms fixed with cone opening  $\phi$ , and tilt both palms clockwise, keeping  $\phi$  fixed, until the right palm is horizontal. The other type of motion was to start with the right palm horizontal, and tilt clockwise until the left palm is horizontal. In Figure 5, each arc is labeled with the  $\phi$  used, and the direction of the tilt.

#### 4.1.1 EXPERIMENTAL RESULTS

A planner to find homing sequences using the frictionless quasistatic assumption was written. The above plan was one of the sequences found for the example object. However, when the plan was tried on the airtable system, it failed regularly. The problem was the frictional instability of the circled state in the third stage of the plan shown in Figure 5. The object would often roll off this edge, into another state not anticipated by the planner.

### 4.2 EXTENSIONS WITH FRICTION

If the assumptions about the system in the previous section are violated, then the object can end up in a state not anticipated by the planner. In terms of the power set graph, this means that an arc out of a particular node may not end up at a node that fully describes the possible states of the system. One reason for this could be the

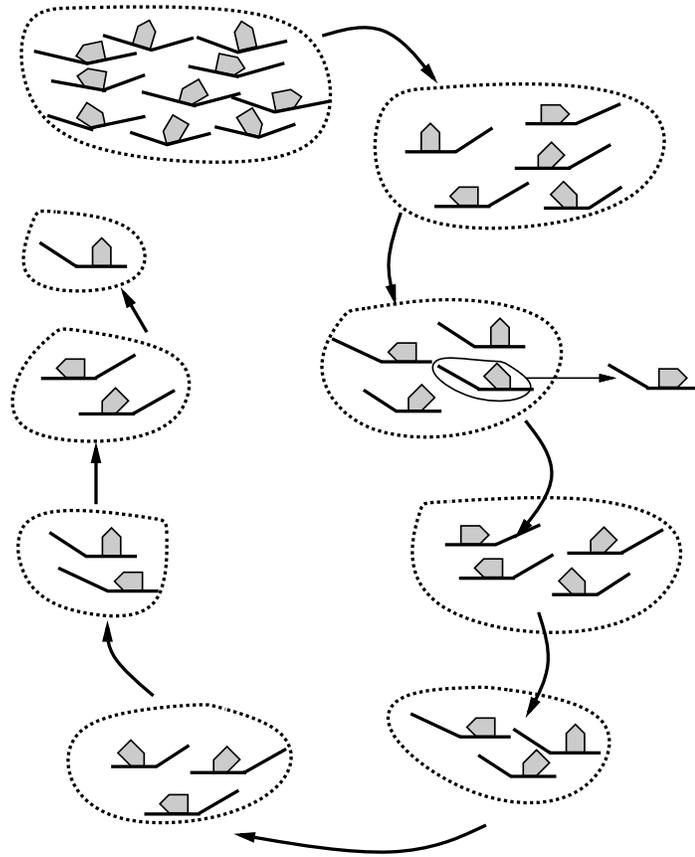


Figure 5: Example homing sequence. The circled configuration in the third stage would often collapse to another state, shown, which was not predicted by the planner.

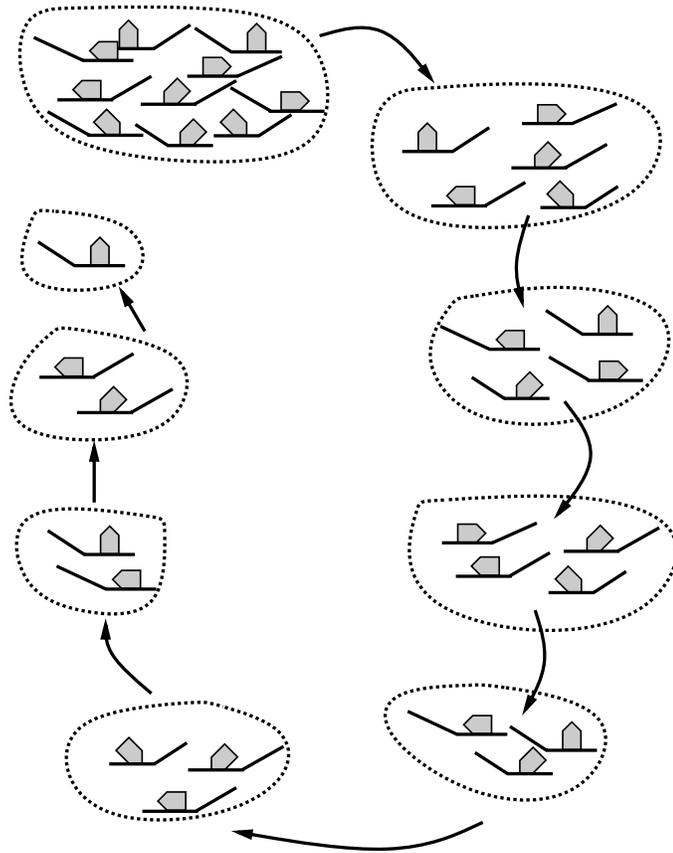


Figure 6: Example homing sequence

frictional instability of a particular nominally stable edge. If one of the edge contacts rolls, rather than slides, the palms may follow a path which is entirely contained in the equivalence region according to the frictionless analysis, and yet the object may transit to another, unpredicted state. This was one of the reasons that the plans generated by the algorithm in the previous section would fail in practice.

Fortunately, frictional instabilities can easily be incorporated into the power set approach, with the additional assumption that the object will roll to the next stable edge and stay there, rather than tumble further beyond to yet another stable edge.

Chapter 4 describes an analysis for identifying edges which are potentially unstable with respect to the contact friction of the system. By knowing which edges will likely have rolling contacts, and in which direction they will roll, we can identify the additional states which may result for a given palm motion. While generating the transition graph for the power set of the object states, we take into account both the frictionless and frictional analyses in determining possible end states. This covers both the possibilities that the object will either slide or roll. We can then search this graph for a homing sequence. Because we incorporate the possibility of both sliding and rolling contacts, the transition graph and any resulting homing sequences will be valid for any coefficient of friction from 0 to  $\mu_0$ . For frictional coefficients higher than  $\mu_0$ , the graph may no longer be valid, since edges which were assumed to always slide may become frictionally unstable.

#### 4.2.1 EXPERIMENTAL RESULTS

The planner was extended to take frictional instability under consideration. Figure 6 shows a homing sequence found for our example object, using the set of cone openings shown to build the arcs. The coefficient of friction,  $\mu_0$ , was taken to be about 0.2. The sequence was run with the object started in all ten of the possible starting conditions, and was successfully brought to the goal state from each initial state. Then the object was dropped into the palms, into an arbitrary initial state, and the sequence was executed. Out of 30 such trials, the object failed to reach the desired goal state 4 times.

This version of the planner can also be used to find frictionally reliable paths from known initial states as well as from unknown ones. Instead of backchaining all the way back to the set of all states, the backchaining terminates upon reaching a node which contains the desired initial state.

### 5 DYNAMICS AND IMPACT

When the velocities of the system are larger than the quasistatic bounds, forces and velocities are no longer in as close correspondence as has been assumed in the previous sections. Centripetal and coriolis effects may begin to be significant. In addition, nonzero relative velocities generate impact forces as contacts are made. We would like conditions on the relative motion of the hand and object such that the hand acquires the object. We would further like the position of the object in the gripper to be known, that is, the part comes to rest in the palm in a predictable position, rather than tumbling to another position.

In Chapter 6, we look at some conservative approximations which can be used to estimate bounds on the manipulator velocity, in order to minimize dynamic effects. In addition to assuming knowledge of the CG and radius of gyration, we use a rigid body impulsive impact model, as described by [3], and used by [7], with the mass of the hand being much greater than the mass of the object, so that the change in velocity due to collision takes place entirely in the motion of the object, and the motion of the hand is unaffected. We also assume that at the moment of impact, the dominant force is the impact force.

### 6 CONCLUSION

We present a model of nonprehensile manipulation, using two one degree of freedom palms, and develop a planning method for part reorientation with our model. Our method finds feasible paths through the space of equivalent state configurations of the object in the palms, without requiring that the palms maintain stable support of the object over the entire path. We show that such a device can reliably orient parts in the plane. This device has demonstrated a number of points.

First, simple low degree of freedom devices can be used for reliable, fast manipulation of objects. In parts orienting scenarios, one would like to avoid complex mechanisms and sensors, which may break down or need careful recalibration. In addition, integrating sensor input into a manipulation algorithm increases the computational complexity, and may slow the action of the device. Sensory feedback is of course necessary in unknown or unstructured environments. However, for tasks in a structured environment, where the same action is repeated continuously, manipulators such as the one studied in this dissertation have a distinct advantage.

Second, because the devices are mechanically simple, the analysis of their mechanics is also relatively simple. To change the task from one object to another, or to change the goal state for the same object, requires only a simple software modification. APOS trays or bowl feeders, which have the same strengths of quick, reliable performance for a given task, must be custom designed to each task, whereas devices such as this one can be used for a variety of tasks. The planner we have designed is efficient, and flexible. Planning reorientations for a variety of objects requires only the geometrical descriptions of the objects: vertices, center of gravity, radius of gyration.

Third, by not relying on force closure grasps, we can exploit gravitational forces to guide the object into the correct state, without excessively precise control over the manipulator motions. Nor do we need extremely precise knowledge of frictional or restitutional coefficients. Rough estimates are sufficient. The primary mechanical analysis used by the planner is frictionless and quasistatic. Knowledge of frictional and dynamic forces is only approximate, yet the resulting plans are robust to initial conditions, friction and to small errors in the calibration of the manipulator.

Other issues such as parts singulation, higher throughput and more general object shape must be addressed in order to make such a device as we have described truly practical. However, we believe that the continued

development of devices such as the one presented in this dissertation is necessary to meet the demands of modern industrial automation.

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